Name ____ Test 2, Fall 2020

1) Find the determinant of the matrix below. (15 points)

$$\begin{bmatrix} 0 & 2 & 0 & 4 \\ 0 & 2 & 0 & -1 \\ 3 & 0 & 6 & 0 \\ 1 & -1 & 1 & 0 \end{bmatrix}$$

$$\begin{vmatrix} 0 & 2 & 0 & 4 \\ 0 & 2 & 0 & -1 \\ 3 & 0 & 6 & 0 \\ 1 & -1 & 1 & 0 \end{vmatrix} = -2\begin{vmatrix} 0 & 0 & -1 \\ 3 & 6 & 0 \\ 1 & 1 & 0 \end{vmatrix} - 4\begin{vmatrix} 0 & 2 & 0 \\ 3 & 0 & 6 \\ 1 & -1 & 1 \end{vmatrix} = -2(-1)\begin{vmatrix} 3 & 6 \\ 1 & 1 \end{vmatrix} - 4(-2)\begin{vmatrix} 3 & 6 \\ 1 & 1 \end{vmatrix}$$

$$= 2(3-6) + 8(3-6) = -6 - 24 = -30$$

2) Given the basis and vector $\vec{x}_B$ below, find $\vec{x}_S$. (10 points)

$$B = \left\{\begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}0\\5\end{bmatrix}\right\} \quad \vec{x}_B = \begin{bmatrix}2\\3\end{bmatrix}$$

$$\vec{x}_S = 2\begin{bmatrix}1\\2\end{bmatrix} + 3\begin{bmatrix}0\\5\end{bmatrix} = \begin{bmatrix}2\\19\end{bmatrix}$$

3) Given the two bases below, find the change of basis matrix that converts information from coordinate vectors in B_2 to coordinate vectors in B_1, denoted by $[I]_{B_2}^{B_1}$. You do not need to perform the arithmetic. (10 points)

$$B_1 = \left\{\begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}0\\2\\2\end{bmatrix}\begin{bmatrix}1\\1\\0\end{bmatrix}\right\} \quad B_2 = \left\{\begin{bmatrix}5\\0\\0\end{bmatrix}, \begin{bmatrix}1\\2\\0\end{bmatrix}, \begin{bmatrix}1\\0\\3\end{bmatrix}\right\}$$

$$[I]_{B_2}^{B_1} = \begin{bmatrix}1 & 0 & 1\\2 & 2 & 1\\3 & 2 & 0\end{bmatrix}^{-1}\begin{bmatrix}5 & 1 & 1\\0 & 2 & 0\\0 & 0 & 3\end{bmatrix}$$

4) Answer the questions below (3 points each)

(A) Let A be a 2×2 matrix with $|A| = 0$. How many solutions does $A\vec{x} = \vec{0}$ have?

∞

(B) Let A be a 2×2 matrix with $|A| = 1$. What is the rank of A?

2

(C) Let A be a 5×7 matrix with $\dim\big(NS(A)\big) = 3$. When row reduced, how many pivots does A have?

4

(D) Let $T: \mathbb{R}^2 \to \mathbb{R}^5$ be a one-to-one linear transformation with corresponding matrix A. When A is row reduced, how many rows of zeroes does it have?

3

(E) Suppose $A\vec{x} = \vec{b}$ is a system of equations that does not have a solution. If A is 4×5, what is the minimum number of free variables in the system of equations.

2

5) Given the system of equations below, use Cramer's Rule to write down a formula for the solution. You do not need to simplify or evaluate your answer(s). (10 points)

$$3x + 2y = 5$$
$$4x - 6y = 7$$

$$x = \frac{\begin{vmatrix} 5 & 2 \\ 7 & -6 \end{vmatrix}}{\begin{vmatrix} 3 & 2 \\ 4 & -6 \end{vmatrix}} \quad y = \frac{\begin{vmatrix} 3 & 5 \\ 4 & 7 \end{vmatrix}}{\begin{vmatrix} 3 & 2 \\ 4 & -6 \end{vmatrix}}$$

The following row reduction may be useful for these problems.

$$\begin{bmatrix} 2 & 3 & 1 \\ 2 & 4 & -2 \\ 1 & 1 & 2 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{bmatrix}$$

6) Are the vectors below linearly indepent or linearly dependent? Justify your answer.
(10 points. 3 for the answer; 7 for the reasoning)

$$\begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}, \text{ and } \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$$

They are linearly dependent. We see from the row reduced matrix that the third column does not have a pivot. That means that the information provided in that vector is redundant.

7) Is the linear transformation below one-to-one? Justify your answer.
(10 points. 3 for the answer; 7 for the reasoning)

$$T(\vec{x}) = \begin{bmatrix} 2 & 3 & 1 \\ 2 & 4 & -2 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

The linear transformation T is not one-to-one. We see from the row reduced matrix that the third column does not have a pivot, meaning that the same linear combinations can be obtained in multiple ways.

8) Find the product below. (10 points)

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 2 & 4 & 5 \\ -1 & 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 4 & 9 \\ 2 & 12 & 23 \end{bmatrix}$$

9) Find length of the vector below. (5 points)

$$\begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$$

$$\left\| \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix} \right\| = \sqrt{9+16+1} = \sqrt{26}$$

10) Given the system of equations below, identify which variable(s) are leading and which variable(s) are free. (5 points)

$$\begin{aligned} 2x \qquad + 3z &= 7 \\ 4y - 8z &= 6 \end{aligned}$$

x and y are leading. z is free.